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COMPUTER TECHNOLOGIES OF SOLUTION OF THE INVERSE PROBLEMS OF THERMOELECTRICITY

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• *The inverse problems of thermoelectricity of eddy currents control are investigated. The finite-element equations for simulation of the temperature fields in different thermoelectric environments and the classification of the inverse problems of thermoelectricity are presented. The algorithm for the numerical solution of the basic inverse problems of thermoelectricity by the finite-element method is described. The examples of simulation of thermoelectric fields for specific geometric areas are realized. Promising directions of the development of the theory of thermoelectric inverse problems are indicated.*

Introduction

In the process of investigating new thermoelectric devices and elements the problem of determination of the initial parameters on the basis of desired distribution of eddy currents arises, which is referred to as the inverse problem of thermoelectricity (IPT) [1]. The most common mathematical model describing thermoelectric effects is a generalized system of Maxwell's equations [2] containing the law of thermoelectric induction [3]. This system of equations is crucial for the research, optimization and improvement of new types of thermocouples.

Each direct problem in the framework of adopted mathematical model can be compared with some set of inverse problems (IP) [4].

In general, the inverse problems for any physical process or technical system can be divided into three groups:

1. The inverse problems arising during the diagnostics and identification of physical processes.
2. The inverse problems arising during the design of technical objects.
3. The inverse problems arising during process and object control.

The first group of the inverse problems associated with experimental investigations, is determined by the need to restore the input (causal) data of some experimental data, i.e. the development of mathematical model based on a series of experiments.

The second group of the inverse problems – i.e. design problems – lies in the definition of design characteristics of the technical object on the basis of given quality coefficients at appropriate limitations.

The third group of IP consists in the study of control actions, by means of which a certain effect of regulation manifested through the system state, is realized.

The inverse problem of thermoelectricity – i.e. the problem of thermoelectric eddy currents control – refers to the third classification group of the inverse problems.

As is known, for the models that are described by equations (systems of equations) in partial derivatives, an analytic solution is rarely found. Even if it is available, the mapping, analysis and interpretation of simulation results is a laborious and not always feasible task. Comprehensive IPT analysis requires appropriate numerical methods, programs that realize the algorithms for IPT solving in general. The most universal and well-developed method, which is also implemented in practice and theoretically grounded one is the finite-element method (FEM). This method is adequately described and implemented for various fields of natural science, and is successfully used in thermoelectricity. In a number of studies [5, 6] the finite-element method to study thermocouples is described and implemented using commercial software packages ANSYS [7] and Comsol Multiphysics [8]. In these works the direct problem of thermoelectricity is investigated, but for the inverse problems the finite-element equations are not derived and there is no specific software.

The objective of this work consists in deriving the finite-element equations for IPT, developing and describing the algorithm of their solution, realizing the computer models for calculation of the temperature fields for various thermoelectric environments.

1. Classification of the inverse problems of thermoelectricity

Most of physical phenomena and processes are characterized by a mathematical model in the form of equations with partial derivatives. The analysis of the generalized system of Maxwell's equations shows that there exists not a single, but a number of IPT having practical importance. In its turn, based on the character of problem statement, the following groups of the inverse problems can be identified [8]:

- *Coefficient-type*. This group of IPT is characterized by unknown coefficients in equations and / or unknown right side of the equation. Coefficient IPT are typical for search problems and designing new materials with predetermined properties and calculation of kinetic coefficients based on a series of physical experiments.
- *Geometrical-type*. Based on given coefficients of equations and physical field it is necessary to find the corresponding geometric area. This type of problem is closely connected with the theory of optimal control and optimization problems. Geometric IPT are applied to find the optimal geometry of the thermocouples and control over thermoelectric fields in the given geometry.
- *Boundary-type*. Boundary IPT are characterized by the absence of direct measurements at the boundary, using available data on certain points within the region. In this case, IPT consists in identifying the boundary conditions and reestablishing cause-and-effect relations. Boundary IPT are typical at investigation and development of sensory-type thermocouples.
- *Evolution IPT*. The inverse problems which are identified by the initial conditions, sometimes they are referred to as retrospective, are the problems of reestablishing the history of the physical process on the basis of consequences.

The basic inverse problem of thermoelectricity consists in the search for temperature field based on given configuration of currents under certain kinetic coefficients and given geometric area.

In general, the basic inverse problem of thermoelectricity is formulated in [1, 10].

We formulate it hereinafter. Let us consider a spatial connected region filled with D medium. The region has material characteristics – tensors of electrical resistance $\hat{\rho}$ and thermopower $\hat{\alpha}$. Tensors are given and dependent on the radius – vector $\mathbf{r}$. In a closed D region the vector field is given $\mathbf{j}(\mathbf{r})$.

The main inverse problem of thermoelectricity is formulated as follows: Based on given current distribution $\mathbf{j}(\mathbf{r})$ in some limited environment with given boundary conditions, which is determined by corresponding material characteristics – tensors $\hat{\rho} = \hat{\rho}(\mathbf{r})$ and, it is necessary to reestablish their generating temperature field $T(\mathbf{r})$ ($\tau(\mathbf{r}) = \nabla T(\mathbf{r})$), including the vector one.

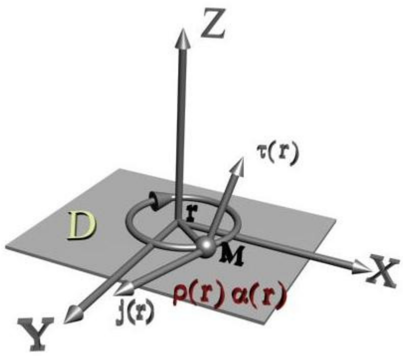


Fig. 1. The inverse problem of thermoelectricity: $\mathbf{r}$ *is the radius-vector of* M *point,*
$\mathbf{j}(\mathbf{r})$ *is the vector function of given configuration of eddy currents,*
$\tau(\mathbf{r})$ *is an unknown vector function of temperature gradients in* D *region.*
$\hat{\rho}(\mathbf{r}),\hat{\alpha}(\mathbf{r})$ *are electrical resistance tensor and thermoelectric power tensor.*

In general case of inhomogeneous and anisotropic environment IPT consists in finding the solution of a system of three differential equations with partial derivatives of the second order:

$$\varepsilon_{ijk}\frac{\partial}{\partial x_j}\alpha_{kl}\frac{\partial}{\partial x_l}T=-J_i\,, \tag{1}$$

where ε_{ijk} is the Levi-Chivita tensor: $\varepsilon_{ijk}=\begin{cases}+1 & P(i,j,k)=1\\ -1 & P(i,j,k)=-1\\ 0 & i=j,j=k,k=i\end{cases}$, $i,j,k,l=\overline{1,3}$, $\mathbf{r}=(x_1,x_2,x_3)$

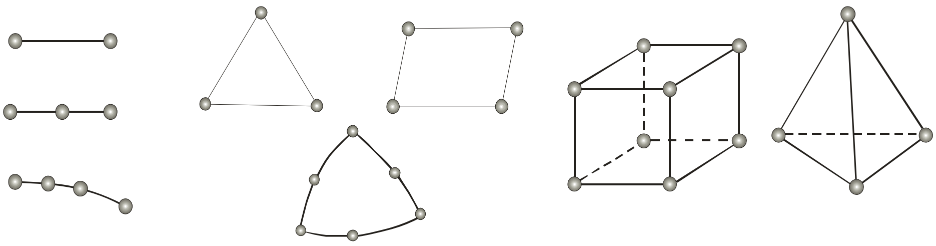
The existence of solution of the system equations (1) depends on the properties of D medium. The solution of the inverse problem of thermoelectricity does not always exist. In [1, 10, 11] IPT is analyzed for different environments as well as the conditions under which a solution exists. Common solutions of IPT are given, as well as analytical solutions for some given configuration of currents in simple geometric areas.

Since the analytic solution is not always possible, and as a rule it is just the case, then for the solution of IPT in an random region, with complex geometry and given spatial configuration of currents, it is necessary to use numerical methods, namely the finite-element method [12], as the most universal modern method of solving physical problems.

2. The finite-element method (FEM) for the inverse problems of thermoelectricity

FEM is described in numerous works, its number exceeds few millions, but among them the works [13, 14] are worth noting, which describe the methods of solution of electromagnetic problems using FEM. Following [1], we consider some of the environments in which the existence of eddy thermoelectric currents is possible, and derive the finite-element equation for the system (1) in each of these environments.

There are several types of the finite-element methods: variation method, method of weighted residuals, vector FEM [15], method of moments. Also one distinguishes the finite-element method in accordance with the type of finite elements, which are used for mathematical model approximation. These types are classified according to the number of nodes on the element and according to geometric shape.



a) b) c)

Fig. 2. Certain types of finite elements: a) one-dimensional, b) two-dimensional, c) three-dimensional.

Galerkin-Petrov method of weighted residuals is the most commonly used [13], which uses direct differential equations in partial derivatives, and problem boundary conditions.

The finite-element method can be divided into stages [16].

- The discretization of geometric area. There are specialized algorithms for triangulation and generation of a finite-element mesh [17]. These algorithms are described in detail in literature.

- The choice of approximation of the solution on a finite element. The choice of an element and approximating function depends on the accuracy of problem solution and the complexity of geometric area. For higher accuracy of problem solution approximating functions of higher-orders are recommended to be used [13]. The most commonly used ones are the Lagrange polynomials.

- The formation of basic functions. For the method of weighted residuals basic functions coincide with the weight ones.

- The calculation of differential problem residual using an approximate solution in the form of series.

- The formation of stiffness matrixes for elements. Residual orthogonalization.

- Ensembling of stiffness matrixes by elements.

- Consideration of boundary conditions.

- Solution of the system of algebraic equations.

Fig. 3 shows the stage of area discretization – i.e. area division into the totality of finite elements. The geometric area, which includes a given field(s), is divided into a set of finite elements $\Omega = \sum_e \Omega^e$, boundary elements $\Gamma = \sum_e \Gamma^e$ are defined separately.

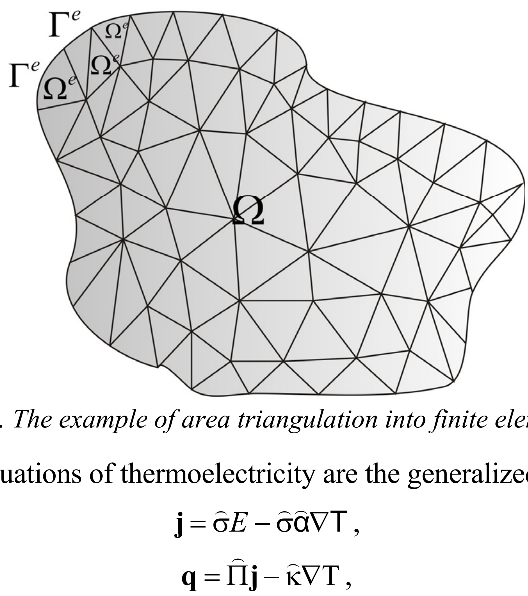


Fig. 3. The example of area triangulation into finite elements.

The main governing equations of thermoelectricity are the generalized Ohm's and Fourier law [1]:

$$\mathbf{j} = \hat{\sigma} E - \hat{\sigma}\hat{\alpha}\nabla T , \qquad (2)$$

$$\mathbf{q} = \hat{\Pi}\mathbf{j} - \hat{\kappa}\nabla T , \qquad (3)$$

where $\mathbf{j}$ is the vector of current density, $\hat{\sigma}$ is the tensor of electrical conductivity, $\hat{\alpha}$ is Seebeck tensor, $\hat{\Pi}$ is Peltier tensor, $\mathbf{q}$ is the vector of heat flux, $\hat{\kappa}$ is the tensor of thermal conductivity, T is the temperature.

Consider the expression for the energy flux density

$$\mathbf{W} = \mathbf{q} + \mu\mathbf{j}\,. \tag{4}$$

In the stationary case

$$\operatorname{div}\mathbf{j} = 0\,, \tag{5}$$

$$\operatorname{div}\mathbf{W} = 0\,, \tag{6}$$

therefore the energy conservation law takes the form:

$$\operatorname{div}\mathbf{q} - (\mathbf{jE}) = 0\,. \tag{7}$$

We calculate the first term of the last equation. It follows from the Fourier law (3), that:

$$\operatorname{div}\mathbf{q} = -\frac{\partial}{\partial x_i}\kappa_{ik}\frac{\partial T}{\partial x_k} + \left(\left(\frac{\partial \Pi_{ik}}{\partial x_i}\right)_T + \frac{\partial \Pi_{ik}}{\partial T}\cdot\frac{\partial T}{\partial x_i}\right)j_k + \Pi_{ik}\frac{\partial j_k}{\partial x_i}\,. \tag{8}$$

Taking into consideration the generalized Ohm's law, we calculate the second term of the energy conservation law:

$$\mathbf{jE} = \rho_{ik} j_k j_i + \alpha_{ik}\frac{\partial T}{\partial x_k} j_i\,. \tag{9}$$

And we come to the law of conservation in the stationary case:

$$-\frac{\partial}{\partial x_i}\kappa_{ik}\frac{\partial T}{\partial x_k} + \left(\frac{\partial \Pi_{ik}}{\partial x_i}\right)_T j_k + \frac{\partial \Pi_{ik}}{\partial T}\cdot\frac{\partial T}{\partial x_i} j_k + \Pi_{ik}\frac{\partial j_k}{\partial x_i} - \rho_{ik} j_k j_i - \alpha_{ik}\frac{\partial T}{\partial x_k} j_i = 0\,. \tag{10}$$

For an inhomogeneous isotropic case the basic equation will have the form:

$$\operatorname{div}(\kappa\nabla T) + \rho j^2 - \tau(\mathbf{j}\nabla T) = 0\,. \tag{11}$$

Taking into consideration the correlation for the Thomson coefficient $\tau = T\dfrac{\partial\alpha}{\partial T}$ when $\alpha \neq \alpha(T)$ we get the following equation

$$\operatorname{div}(\kappa\nabla T) + \rho j^2 - T\nabla\alpha\mathbf{j} = 0\,. \tag{12}$$

For simplicity, we consider two-dimensional case. We write this equation in the component form

$$\frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) + \rho\left(j_x^2 + j_y^2\right) - T\left(j_x\nabla_x\alpha + j_y\nabla_y\alpha\right) = 0\,. \tag{13}$$

We divide the area into the elements Ω^e. We define the approximating function on each element – i.e. the basic function in the form $N_{ij}^e = \Lambda_i^p(x)\Lambda_j^p(y)$, where (i, j) are the indexes of mesh point, Λ^p are the fundamental Lagrange polynomials of p degree:

$$\begin{aligned}\Lambda_l^p(x) = &\left[(x - x_0)(x - x_1)\ldots(x - x_{l-1})(x - x_{l+1})\ldots(x - x_p)\right]\times\\ &\times\left[(x_l - x_0)(x_l - x_1)\ldots(x_l - x_{l-1})(x_l - x_{l+1})\ldots(x_l - x_p)\right].\end{aligned} \tag{14}$$

We denote $g = \rho\mathbf{j}^2$ and $f = -\nabla\alpha\mathbf{j}$ and consider the residual

$$R^e = \frac{\partial}{\partial x}\left(\kappa\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) + fT + g\,. \tag{15}$$

To find the unknown temperature $R^e \to \min_{\Omega^e}$ should be minimized:

We introduce the weight functions and integrate the residual by the element area:

$$\iint_{\Omega^e} W\left[\frac{\partial}{\partial x}\left(\kappa_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\kappa_y \frac{\partial T}{\partial y}\right)+fT+g\right]dxdy=0\,. \tag{16}$$

Taking into consideration $\frac{\partial}{\partial x}\left(W\kappa_x \frac{\partial T}{\partial x}\right)=\frac{\partial W}{\partial x}\left(\kappa_x \frac{\partial T}{\partial x}\right)+W\frac{\partial}{\partial x}\left(\kappa_x \frac{\partial T}{\partial x}\right)$, we obtain the following relation for the residual

$$\begin{gathered}\iint_{\Omega^e}\left[\frac{\partial}{\partial x}\left(W\kappa_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(W\kappa_y \frac{\partial T}{\partial y}\right)\right]dxdy-\\ -\iint_{\Omega^e}\left[\kappa_x \frac{\partial W}{\partial x}\frac{\partial T}{\partial x}+\kappa_y \frac{\partial W}{\partial y}\frac{\partial T}{\partial y}\right]dxdy+\iint_{\Omega^e} fWTdxdy=-\iint_{\Omega^e} gWdxdy.\end{gathered} \tag{17}$$

Using Green's theorem for the first integral, we obtain

$$\iint_{\Omega^e}\left[\frac{\partial}{\partial x}\left(W\kappa_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(W\kappa_y \frac{\partial T}{\partial y}\right)\right]dxdy=\oint W\left(\kappa_x \frac{\partial T}{\partial x}n_x+\kappa_y \frac{\partial T}{\partial y}n_y\right)dl\,, \tag{18}$$

where n_x, n_y are the components of the normal vector to the surface of the element.

Substituting (18) in (17) we obtain the following equation

$$-\iint_{\Omega^e}\left[\kappa_x \frac{\partial W}{\partial x}\frac{\partial T}{\partial x}+\kappa_y \frac{\partial W}{\partial y}\frac{\partial T}{\partial y}\right]dxdy+\iint_{\Omega^e} fWTdxdy=-\iint_{\Gamma^e} Wgdxdy-\iint_{\Omega^e} W\left(\kappa_x \frac{\partial T}{\partial x}n_x+\kappa_y \frac{\partial T}{\partial y}n_y\right)dl\,. \tag{19}$$

According to Galerkin's method basic functions are chosen as weight functions, which approximate an unknown quantity, in this case the temperature T

$$T=\sum_{j=1}^{n} T_j^e N_j\,, \tag{20}$$

where N_j are corresponding basic functions, which are presented in the form of Lagrange polynomials, T_j^e are the temperature values in the nodes of the element, n is the number of nodes of the element.

Thus, $W=N_i$, $i=\overline{1,n}$, by substituting the weight function in equation (19) we obtain the system of equations:

$$\begin{gathered}-\iint_{\Omega^e}\left[\kappa_x \frac{\partial N_i}{\partial x}\frac{\partial\left(\sum_{j=1}^{n} T_j^e N_j\right)}{\partial x}+\kappa_y \frac{\partial N_i}{\partial y}\frac{\partial\left(\sum_{j=1}^{n} T_j^e N_j\right)}{\partial y}\right]dxdy+\iint_{\Omega^e} gN_i\left(\sum_{j=1}^{n} T_j^e N_j\right)dxdy=\\ =\iint_{\Omega^e} N_i gdxdy-\oint_{\Gamma^e} N_i\left(\kappa_x \frac{\partial T}{\partial x}n_x+\kappa_y \frac{\partial T}{\partial y}n_y\right)dl,\quad i=\overline{1,n}.\end{gathered} \tag{21}$$

The last contour integral must satisfy the boundary conditions of the problem. The system can be rewritten as

$$-\iint_{\Omega^e}\left[\kappa_x\left(\frac{\partial N_i}{\partial x}\right)\sum_{j=1}^{n}T_j^e\frac{\partial N_j}{\partial x}+\kappa_y\left(\frac{\partial N_i}{\partial y}\right)\sum_{j=1}^{n}T_j^e\frac{\partial N_j}{\partial y}\right]dxdy+\iint_{\Omega^e}gN_i\left(\sum_{j=1}^{n}T_j^eN_j\right)dxdy=$$
$$=\iint_{\Omega^e}N_igdxdy-\oint_{\Gamma^e}N_i\left(\kappa_x\frac{\partial T}{\partial x}n_x+\kappa_y\frac{\partial T}{\partial y}n_y\right)dl,\quad i=\overline{1,n}. \tag{22}$$

This system can be represented in the matrix form:

$$\begin{bmatrix} M_{11}^e & M_{12}^e & \dots & M_{1n}^e \\ M_{21}^e & M_{22}^e & \dots & M_{2n}^e \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1}^e & M_{n2}^e & \dots & M_{nn}^e \end{bmatrix}\begin{Bmatrix} T_1^e \\ T_2^e \\ \vdots \\ T_n^e \end{Bmatrix}+\begin{bmatrix} A_{11}^e & A_{12}^e & \dots & A_{1n}^e \\ A_{21}^e & A_{22}^e & \dots & A_{2n}^e \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1}^e & A_{n2}^e & \dots & A_{nn}^e \end{bmatrix}\begin{Bmatrix} T_1^e \\ T_2^e \\ \vdots \\ T_n^e \end{Bmatrix}=\begin{Bmatrix} s_1^e \\ s_2^e \\ \vdots \\ s_n^e \end{Bmatrix}+\begin{Bmatrix} p_1^e \\ p_2^e \\ \vdots \\ p_n^e \end{Bmatrix}, \tag{23}$$

where

$$M_{ij}^e=-\iint_{\Omega^e}\left[\kappa_x\left(\frac{\partial N_i}{\partial x}\right)\left(\frac{\partial N_j}{\partial x}\right)+\kappa_y\left(\frac{\partial N_i}{\partial y}\right)\left(\frac{\partial N_j}{\partial y}\right)\right]dxdy\,, \tag{24}$$

$$A_{ij}^e=\iint_{\Omega^e}gN_iN_jdxdy\,, \tag{25}$$

$$s_i^e=\iint_{\Omega^e}N_igdxdy \text{ и } p_i^e=-\oint_{\Gamma^e}N_i\left(\kappa_x\frac{\partial T}{\partial x}n_x+\kappa_y\frac{\partial T}{\partial y}n_y\right)dl\,. \tag{26}$$

In a more compact form, the system has the form

$$\begin{bmatrix} K_{11}^e & K_{12}^e & \cdots & K_{1n}^e \\ K_{21}^e & K_{22}^e & \cdots & K_{2n}^e \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1}^e & K_{n2}^e & \cdots & K_{nn}^e \end{bmatrix}\begin{Bmatrix} T_1^e \\ T_2^e \\ \vdots \\ T_n^e \end{Bmatrix}=\begin{Bmatrix} b_1^e \\ b_2^e \\ \vdots \\ b_n^e \end{Bmatrix},\qquad \text{where}\ \begin{matrix} K_{ij}^e=M_{ij}^e+A_{ij}^e \\ b_i^e=s_i^e+p_i^e \end{matrix} \tag{27}$$

Thus, we achieved a system of nonlinear equations on the element. Next, the ensembling procedure is conducted, i.e. the summation of the each element contribution in the system. After satisfying the boundary conditions the system of nonlinear equations is solved by means of one of the methods for solving dispersed matrixes.

The final stage of simulation is the visualization of thermoelectric fields and the analysis of numerical experiment results. Special algorithms for rendering thermoelectric fields on the basis of [17 – 18] are developed.

The finite-element methods are theoretically grounded, have good precision and are used in many fields of natural science. For thermoelectric analysis these methods are the cornerstone of numerical simulation and theoretical study of thermoelectric phenomena and processes.

3. The example of solving the basic inverse problem of thermoelectricity for inhomogeneous environment in the two-dimensional case

We consider the examples of solution of the basic inverse problem for inhomogeneous isotropic environment.

As geometric region we take the ellipse. Assume that the gradients of thermopower and resistivity are directed along the axis *x*.

$$\alpha(x)=\alpha_0+k_\alpha x\,, \tag{28}$$

$$\rho(x) = \rho_0 + k_\rho x \ . \tag{29}$$

Define the desired configuration of eddy currents using the Lukosz function [19]

$$H(x, y) = a^2x^2 + b^2y^2 - R^2 , \tag{30}$$

where $a, b, R = \text{const}$.

An exact analytical solution is known from [10]:

$$T(x, y) = T_0 + \frac{2}{k_\alpha}\left(\left(a^2 + b^2\right)\rho_0 + \left(2a^2 + b^2\right)k_\rho x\right)y \ . \tag{31}$$

Fig. 4 shows the numerical solution of this problem. The solution coincides with the analytical one up to a passive temperature field.

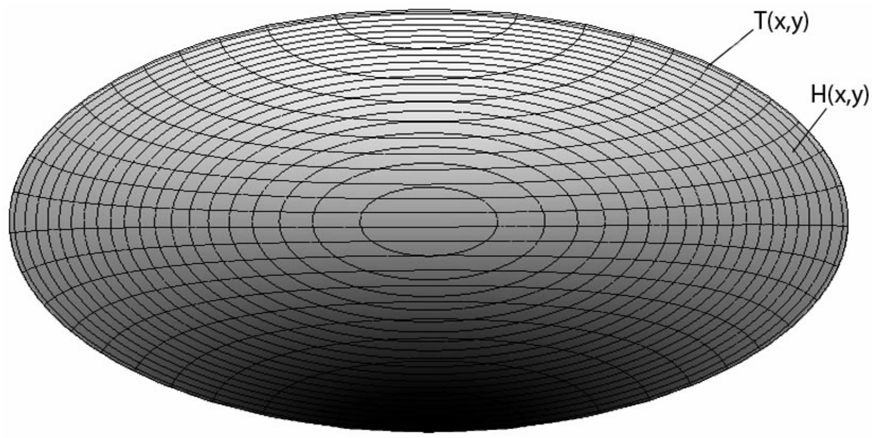


Fig. 4. Thermoelectric eddy currents in the ellipse:
$H(x, y)$ are the current streamlets, $T(x, y)$ is the temperature field with the isotherms.

After a series of computational experiments, it was found that the solution of the problem strongly depends on the kinetic coefficients. Using the coefficients far from real physical values, the solution does not coincide. The issues of stability and correctness of the inverse problems are studied in [20], and of the finite-element method in [21], in particular.

4. The example of solving the basic inverse problem of thermoelectricity for inhomogeneous environment in the three-dimensional case

As geometric region we consider a cylinder with a cut-out core. Generated finite-element mesh is shown in Fig. 5.

Material specifications are given in the form

$$\alpha(x) = \alpha_0 + k_\alpha x , \tag{32}$$

$$\rho(x) = \rho_0 + k_\rho x \ . \tag{33}$$

The configuration of currents is provided in the form of cylindrical spirals

$$\begin{aligned} j_x &= -Cy, \\ j_y &= Cx, \\ j_z &= D, \end{aligned} \tag{34}$$

where C, D are the constants. The corresponding configuration of currents is shown in Fig. 6.

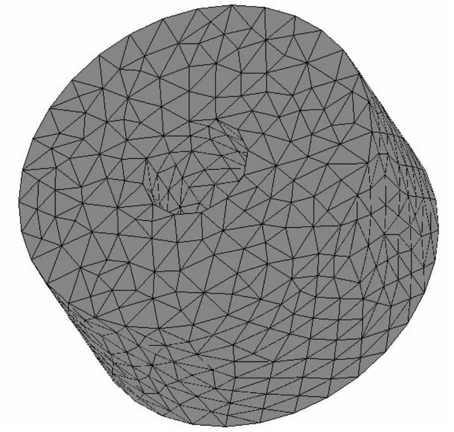


Fig. 5. Finite-element mesh for a cylinder with a cut-out core.

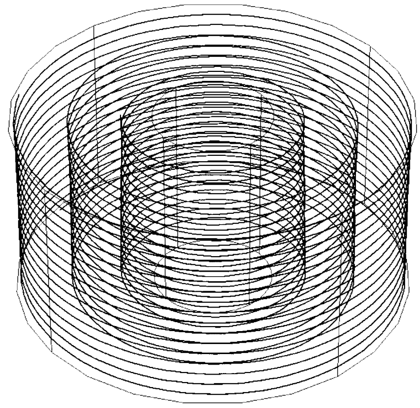


Fig. 6. The spatial configuration of eddy currents in the form of cylindrical spirals.

Fig. 7 shows the calculated temperature field with isothermal planes. It should be noted that the solution of the basic inverse problem, as shown in Fig. 7, is not the only one, other solutions differ from the calculated passive component of the temperature field. The passive part has no effect on the generation of thermoelectric eddy currents.

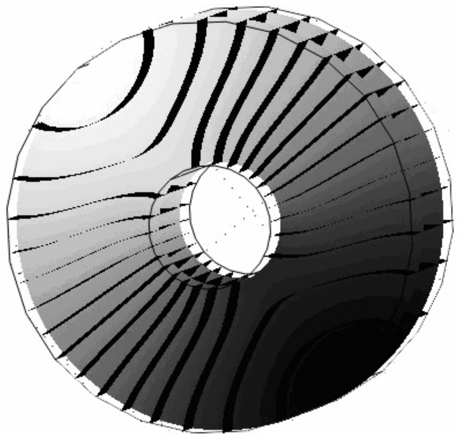


Fig. 7. The temperature distribution in the cylinder with isothermal planes.

Consider the examples of the basic inverse problem in the case of a homogeneous anisotropic environment.

5. The example of solving the basic inverse problem of thermoelectricity for the homogeneous anisotropic environment in the two-dimensional case

As an example, we take a rectangular plate. We define the temperature dependences of kinetic coefficients for *CdSb* [22]:

In order to specify the configuration of currents in the plate we use the vector potential $\mathbf{H}(x, y)$ in the form of a spiral of Archimedes:

$$\mathbf{H} = \sqrt{x^2 + y^2} + \frac{\operatorname{arctg}(y/x)}{2\pi}. \qquad (35)$$

Fig. 8 shows the current streamlets in an anisotropic plate. In the case of anisotropic environment kinetic coefficients become the tensors of the second rank.

$$\hat{\alpha}=\begin{bmatrix}\alpha_{xx} & \alpha_{xy}\\ \alpha_{yx} & \alpha_{yy}\end{bmatrix},\ \hat{\sigma}=\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{yx} & \sigma_{yy}\end{bmatrix},\ \hat{\kappa}=\begin{bmatrix}\kappa_{xx} & \kappa_{xy}\\ \kappa_{yx} & \kappa_{yy}\end{bmatrix}. \quad (36)$$

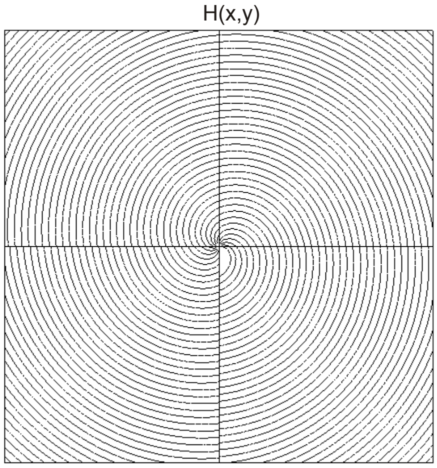


Fig. 8. Thermoelectric eddy currents in an anisotropic plate: $H(x,y)$ are the current streamlets.

In two-dimensional case of anisotropic environment the equation (10) takes the form:

$$\frac{\partial}{\partial x_i}\kappa_{ik}\frac{\partial T}{\partial x_k}+\rho_{ik}j_k j_i-\left(T\alpha_{ik}\right)\frac{\partial j_k}{\partial x_i}-\left(\frac{\partial\left(T\alpha_{ki}\right)}{\partial T}-\alpha_{ik}\right)j_k=0. \quad (37)$$

Active part of the temperature field has the form [1]:

$$T(x,y)=\frac{\rho_{yy}\int j_y dy+\rho_{xx}\int j_x dx}{\alpha_{yy}-\alpha_{xx}}. \quad (38)$$

Analytical solution of the basic inverse problem [10]:

$$T(x,y)=-\frac{1}{2}\frac{\ln\left(\rho_{xx}y+\sqrt{\rho_{xx}x^2+\rho_{yy}y^2}\right)x+\ln\left(\rho_{xx}x+\sqrt{\rho_{xx}x^2+\rho_{yy}y^2}\right)}{\alpha_{xx}-\alpha_{yy}}. \quad (39)$$

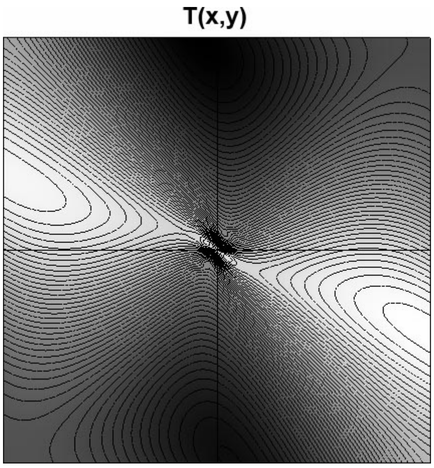


Fig. 9. The temperature field with the isotherms for the excitation of eddy currents in the form of the spirals of Archimedes.

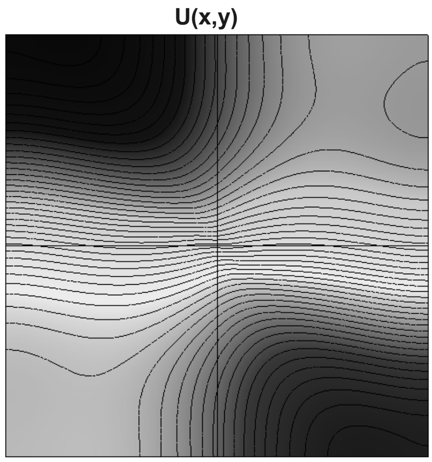


Fig. 10. The potential distribution and equipotential lines of electric field for the excitation of eddy currents in an anisotropic environment.

Fig. 9 shows the calculated temperature field in Comsol Multyphisics package, and Fig. 10

demonstrated the distribution of electric potential. The numerical solution of the problem coincides with the analytical solution.

6. The example of solving the basic inverse problem for the homogeneous anisotropic environment in the three-dimensional case

Similar to the preceding paragraph, the kinetic coefficients are considered for *CdSb*. The geometric region is preset in the form of a sphere. The configuration of thermoelectric eddy currents is given in the form of cylindrical spirals:

$$j_x = -Cy, \quad j_y = Cx, \quad j_z = D, \tag{40}$$

where C, D are the constant. The corresponding configuration of currents is shown in Fig. 11. Fig. 12 shows the solution of the basic inverse problem for anisotropic thermoelectric environment.

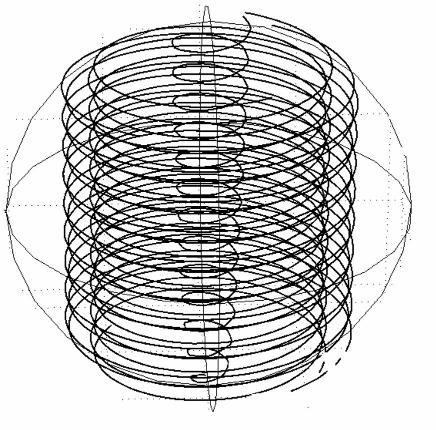


Fig. 11. The spatial configuration of eddy currents in an anisotropic environment in the form of cylindrical spirals.

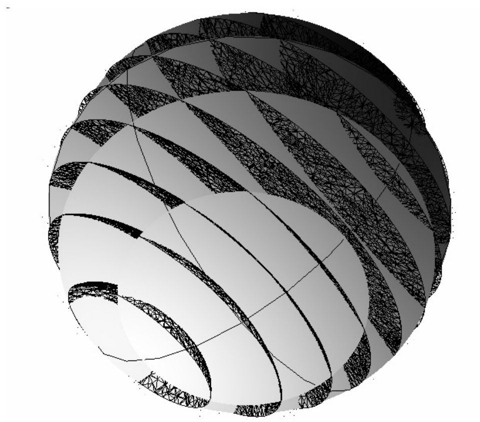


Fig. 12. The temperature field in the sphere with isothermal planes.

The solution of the basic inverse problem of thermoelectricity makes it possible to develop new types of thermocouples and enhance the effectiveness of the existing ones.

Conclusions

1. The classification of the inverse problems of thermoelectricity and the fields of application of IPT for thermoelectric instrument engineering are given.
2. The finite-element equations for numerical simulation of the inverse problems of thermoelectricity for isotropic inhomogeneous and anisotropic thermoelectric environments are derived.
3. The algorithm of computer simulation of IPT was developed and described, the problem of convergence and further improvement of the algorithms for IPT solution is studied.
4. The computer model examples of calculation of temperature fields for various thermoelectric environments, geometric regions and a given configuration of eddy currents are realized.
5. The ways of IPT application to the study and optimization of new types of thermocouples and thermoelectric devices based on them are analyzed.

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